4.103 Exercise and sleep. Suppose 40% of adults get enough sleep, 46% get enough exercise, and 24% do both. Find the probabilities of the following events:

(a) enough sleep and not enough exercise  
(b) not enough sleep and enough exercise  
(c) not enough sleep and not enough exercise  
(d) for each of parts (a), (b), and (c), state the rule that you used to find your answer.

**Solution**

E = Adults that get enough sleep  
F = Adults that get enough exercise

P(E) = 0.40  
P(F) = 0.46  
P(E \cap F) = 0.24

(a) \( P(E \cap F^c) + P(E \cap F) = P(E) \Rightarrow P(E \cap F^c) = P(E) - P(E \cap F) = 0.40 - 0.24 = 0.16 \)

(b) \( P(E^c \cap F) + P(E \cap F) = P(F) \Rightarrow P(E^c \cap F) = P(F) - P(E \cap F) = 0.46 - 0.24 = 0.22 \)

(c) \( P(E^c \cap F^c) = P[(E \cup F)^c] = 1 - P(E \cup F) = 1 - (0.40 + 0.46 - 0.24) = 0.38 \)

(d) Low of the complement in combination with addition rule

Using a two-way table, it's easier to see these relations.

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>E^c</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>P(E \cap F) = 24%</td>
<td>P(E^c \cap F) = 22%</td>
<td>46% = P(F)</td>
</tr>
<tr>
<td>F^c</td>
<td>P(E \cap F^c) = 16%</td>
<td>P(E^c \cap F^c) = 38%</td>
<td>34% = P(F^c)</td>
</tr>
<tr>
<td>Total</td>
<td>P(E) = 40%</td>
<td>P(E^c) = 60%</td>
<td>100%</td>
</tr>
</tbody>
</table>

4.104 Exercise and sleep. Refer to the previous exercise. Draw a Venn diagram showing the probabilities for exercise and sleep.
4.105 Lying to a teacher. Suppose that 48% of high school students would admit to lying at least once to a teacher during the past year and 25% of students are male and would admit to lying at least once to a teacher during the past year. Assume that 50% of the students are male. What is the probability that a randomly selected student is either male or is a liar. Be sure to show your work and indicate all of the rules that you use to find your answer.

Solution

L = A student admits of having lied at least once to a teacher
M = A student is a Male student

We are given:

\[ P(L) = 0.48 \]
\[ P(M) = 0.50 \]
\[ P(L ∩ M) = 0.25 \]

We are asked to find:

\[ P(L \text{ or } M) = P(L) + P(M) - P (L ∩ M) \]
\[ = 0.48 + 0.50 - 0.25 = 0.73 \]

4.106 Lying to a teacher. Refer to the previous exercise. Suppose that you select a student from the subpopulation of liars. What is the probability that the student is female? Be sure to show your work and indicate all of the rules that you use to find your answer.

Solution

F = A student is a Female
\[ P(F) = 0.50 \]
\[ P(L ∩ F) + P(L∩ M) = 0.48 \Rightarrow P(L∩F) = 0.48 - 0.25 = 0.23 \]

We are asked to compute \( P(F |L) \)

\[ P(F |L) = \frac{P(L∩ F)}{P(L)} = \frac{0.23}{0.48} = 0.4792 \]

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>F</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>P(L ∩ M) = 25%</td>
<td>P(L ∩ F) = 23%</td>
<td>P(L) = 48%</td>
</tr>
<tr>
<td>notL</td>
<td>P(M ∩ notL) = 25%</td>
<td>P(notL ∩ F) = 27%</td>
<td>P(notL) = 52%</td>
</tr>
<tr>
<td>Total</td>
<td>P(M) = 50%</td>
<td>P(F) = 50%</td>
<td>100%</td>
</tr>
</tbody>
</table>
4.107 Binge drinking and gender. In a college population, students are classified by gender and whether or not they are frequent binge drinkers. Here are the probabilities:

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binge drinker</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>Not binge drinker</td>
<td>0.32</td>
<td>0.45</td>
</tr>
</tbody>
</table>

(a) Verify that the sum of the probabilities is 1.
(b) What is the probability that a randomly selected student is not a binge drinker?
(c) What is the probability that a randomly selected male student is not a binge drinker?
(d) Explain why your answers to (b) and (c) are different. Use language that would be understood by someone who has not studied the material in this chapter.

**Solution**

Use this table to answer the questions in this exercise.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binge drinker</td>
<td>0.11</td>
<td>0.12</td>
<td>0.23</td>
</tr>
<tr>
<td>Not binge drinker</td>
<td>0.32</td>
<td>0.45</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.43</td>
<td>0.57</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) \( P(\text{Not a binge drinker}) = P(\text{Male and Not Binge Drinker}) + P(\text{Female and Not Binge Drinker}) \)
\[ = 0.32 + 0.45 = 0.77 \]

(c) \( P(\text{Not a binge drinker | Male}) = \frac{P(\text{Not a binge drinker And male})}{P(\text{Male})} \)
\[ = \frac{(0.32)}{(0.43)} = 0.7442 \]

(d) In part (b) we computed the probability that a random person (male or female) is a binge drinker. In part (c) we computed the probability that a random male person is a binge drinker. The two probabilities are different because the two populations from where a person is selected are different.
4.108 Find some probabilities. Refer to the previous exercise.

(a) Find the probability that a randomly selected student is a male binge drinker, and find the probability that a randomly selected student is a female binge drinker.

(b) Find the probability that a student is a binge drinker, given that the student is male and find the probability that a student is a binge drinker, given that the student is female.

(c) Your answer for part (a) gives a higher probability for females, while your answer for part (b) gives a higher probability for males. Interpret your answers in terms of the question of whether there are gender differences in binge-drinking behavior. Decide which comparison you prefer and explain the reasons for your preference.

Solution

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binge drinker</td>
<td>0.11</td>
<td>0.12</td>
<td>0.23</td>
</tr>
<tr>
<td>Not binge drinker</td>
<td>0.32</td>
<td>0.45</td>
<td>0.77</td>
</tr>
<tr>
<td>Total</td>
<td>0.43</td>
<td>0.57</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) \( P(\text{male binge drinker}) = P(\text{binge drinker And male}) = 0.11 \)
\( P(\text{female binge drinker}) = P(\text{binge drinker And female}) = 0.12 \)

(b) \( P(\text{binge drinker | male}) = 0.11/0.43 = .2558 \)
\( P(\text{binge drinker | female}) = 0.12/0.57 = .2105 \)

(c) Because there are more women students than men in the population, a larger % of all students are women binge drinkers. This does not mean that among all women the % of binge drinkers is larger than the % of binge drinkers among all men.

4.113 Find a conditional probability. In the setting of the previous exercise, what is the conditional probability that a household is prosperous, given that it is educated? Explain why your result shows that events \( A \) and \( B \) are not independent.

4.115 Sales of cars and light trucks. Motor vehicles sold to individuals are classified as either cars or light trucks (including SUVs) and as either domestic or imported. In a recent year, 69% of vehicles sold were light trucks, 78% were domestic, and 55% were domestic light trucks. Let \( A \) be the event that a vehicle is a car and \( B \) the event that it is imported. Write each of the following events in set notation and give its probability.

(a) The vehicle is a light truck.
(b) The vehicle is an imported car.
4.117 Conditional probabilities and independence. Using the information in Exercise 4.115, answer these questions.

(a) Given that a vehicle is imported, what is the conditional probability that it is a light truck?
(b) Are the events “vehicle is a light truck” and “vehicle is imported” independent? Justify your answer.