General Addition & Multiplication Rules & Conditional Probabilities

**General Addition Rule**

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

**Conditional Probability Formulas**

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} \quad P(B|A) = \frac{P(A \cap B)}{P(A)} \]

**General Multiplication Rule**

\[ P(A \cap B) = P(A) \cdot P(B|A) = P(B) \]

**Total Probability**

\[ P(B) = P(B \cap A) + P(B \cap notA) = P(B|A) \cdot P(A) + P(B|notA) \cdot P(notA) \]

**Bayes' Formula**

\[ P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} = \]

\[ = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|notA) \cdot P(notA)} \]
Example 1
A recent poll asked a group of registered voters the following questions:

(A) Do you agree of the President's handling of the Iran conflict?
(B) Will you vote for the President's re-election in November?

Of those asked, 60% answered yes to the first question, 48% said they'd vote for his re-election, and 38% answered yes to both of the questions.

Compute the following probabilities:

1. What percent of those asked answered Yes to at least one of the questions?
2. What percent of those asked answered No to both of the questions?
3. What percent of those asked answered Yes to the Iran question only?
4. What percent of those asked answered Yes to the President's re-election question only?
5. What percent of those asked answered Yes to at least one of the questions?
6. Given a person answered Yes to the Iran question, what is the probability the person answered Yes to the re-election question?
7. Given a person answered Yes to the re-election question, what is the probability the person answered No to the Iran question?
8. Given a person answered No to the Iran question, what is the probability the person answered No to the re-election question?

A. Using a Venn Diagram to Answer Probability Questions
Start by drawing a Venn diagram. 'S' represents the entire group, 'A' those who answered Yes to the Iran question, and 'B' those who answered Yes to the re-election question. The overlap of A & B represents those who answered Yes to both the questions.
Use the information given in the problem to come up with the percentages shown in the four areas of the diagram. The problem should give you enough information to be able to do that.

These four areas represent:

38\% = P(A \cap B) = Group that answered yes to both.

22\% = P(A \cap \text{not } B) = P(A) - P(A \& B) .... Group that answered yes only to the Iran question.

10\% = P(\text{not } A \cap B) = P(B) - P(A \& B) .... Group that answered Yes to the re-election question only.

30\% = P(\text{not } A \cap \text{not } B) = 100\% - (38\% + 22\% + 10\%) .... Group that answered No to both questions.

Using these four percentages we can find the probability of any event that is a combination of the four events listed above.

**Answering the questions**

1. Yes to at least one of the questions.

   \[ A \cup B \]
   
   \[ P(A \text{ or } B) = 22\% + 38\% + 10\% = 70\% \]

2. No to both of the questions.

   \[ \text{not } A \cap \text{not } B \]
   
   \[ P(\text{not } A \cap \text{not } B) = P(\text{not}(A \cup B)) = 30\% \]

3. Yes to the Iran question only.

   \[ \text{Only } A = A \cap \text{not } B \]
   
   \[ P(\text{only } A) = 38\% \]

4. Yes to the President's re-election question only.

   \[ \text{Only } B = B \cap \text{not } A \]
   
   \[ P(\text{only } B) = 10\% \]
5. No to at least one of the questions.

\[ \text{not } A \cup \text{not } B = \text{not } (A \cap B) \]

\[
P(\text{not } A \cap B) = 1 - P(A \cap B) = 1 - 0.38 = 0.62 \text{ or } 62\%
\]

6. Yes to the re-election, given Yes to the Iran.

\[
P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.38}{0.60} = 0.6333 \text{ or } 63.3\%
\]

7. No to the Iran, given Yes to the re-election.

\[
P(\text{not } A | B) = \frac{P(\text{not } A \cap B)}{P(B)} = \frac{0.10}{0.48} = 0.2083 \text{ or } 20.83\%
\]

8. No to the re-election, given No to the Iran.

\[
P(\text{not } B | \text{not } A) = \frac{P(\text{not } A \cap \text{not } B)}{P(\text{not } A)} = \frac{0.30}{0.40} = 0.75 \text{ or } 75\%
\]

### B. Using Two Way Tables to Answer Probability Questions

Note: The given percentages in the problem are shown in red.

<table>
<thead>
<tr>
<th></th>
<th>A Yes to Iran</th>
<th>not A No to Iran</th>
<th>Row totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>B Yes to re-election</td>
<td>38%</td>
<td>10%</td>
<td>48%</td>
</tr>
<tr>
<td>not B No to re-election</td>
<td>22%</td>
<td>30%</td>
<td>52%</td>
</tr>
<tr>
<td>Column totals</td>
<td>60%</td>
<td>40%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Start by placing the given probabilities in the appropriate cells of the table. For example, 38% goes at the intersection of column A (Yes to Iran) and row B (Yes to re-election). The total for column A (60%)
represents the percent of those that answered Yes to the Iran question, and the total for row B (48%) represents the percent of those that answered Yes to the re-election question.

Use the fact that the sum of the percentages for any event and its complement must equal 100%, and that the percentages inside the table need to add up to the row and column totals to fill in the rest of the table.

**Answering the questions**

1. What percent of those asked answered Yes to at least one of the questions?
   
   \[ P(\text{Yes to both}) + P(\text{Yes to Iran and no to re-election}) + P(\text{Yes to re-election and no to Iran}) = 38\% + 22\% + 10\% = 70\% \]

2. What percent of those asked answered No to both of the questions?
   
   \[ P(\text{not to both}) = 30\% \]

3. What percent of those asked answered Yes to the Iran question only?
   
   \[ P(\text{Yes to Iran & not to re-election}) = 22\% \]

4. What percent of those asked answered Yes to the President’s re-election question only?
   
   \[ P(\text{Yes to re-election & not to Iran}) = 10\% \]

5. What percent of those asked answered No to at least one of the questions?
   
   \[ P(\text{Not to both}) + P(\text{Not to Iran & Yes to re-election}) + P(\text{Yes to Iran & No to re-election}) = 30\% + 10\% + 22\% = 62\% \]

6. Given a person answered Yes to the Iran question, what is the probability the person answered Yes to the re-election question?
   
   Given that the person answered Yes to the Iran question reduces the table to the first column only (i.e., the 60% who answered Yes to the Iran question).

<table>
<thead>
<tr>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes to Iran</td>
</tr>
<tr>
<td><strong>B</strong></td>
</tr>
<tr>
<td>Yes to re-election</td>
</tr>
<tr>
<td>not B</td>
</tr>
<tr>
<td>No to re-election</td>
</tr>
<tr>
<td>Column totals</td>
</tr>
</tbody>
</table>

   Of those, 38\%/60\% = 63.3\% answered Yes to the re-election question.
7. Given a person answered Yes to the re-election question, what is the probability the person answered No to the Iran question?

Given that the person answered Yes to the re-election question reduces the table to the first row only (i.e., the 48% who answered Yes to the re-election question).

<table>
<thead>
<tr>
<th></th>
<th>A Yes to Iran</th>
<th>not A No to Iran</th>
<th>Row totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>B Yes to re-election</td>
<td>38%</td>
<td>10%</td>
<td>48%</td>
</tr>
</tbody>
</table>

Of those, 10%/48% = 20.83% answered No to the Iran question.

8. Given a person answered No to the Iran question, what is the probability the person answered No to the re-election question?

Given a person answered No to the Iran question reduces the table to the second column only (i.e., those who answered No to the Iran question).

<table>
<thead>
<tr>
<th></th>
<th>not A No to Iran</th>
</tr>
</thead>
<tbody>
<tr>
<td>B Yes to re-election</td>
<td>10%</td>
</tr>
<tr>
<td>not B No to re-election</td>
<td>30%</td>
</tr>
<tr>
<td>Column totals</td>
<td>40%</td>
</tr>
</tbody>
</table>

Of those, 30%/40% = 75% answered no to the re-election question.
Example 2
A recent poll asked a group of 200 men and 300 women whether they approve of President's Obama new health care law. 80% of the women and 70% of the men said they approved of the President's new health care law.

Choose a person at random from the 500 that participated in the survey:

1. What is the probability the person is a woman?
2. What is the probability the person is a man?
3. What is the probability the person approves of the President's new health law?
4. What is the probability the person disapproves of the President's new health law?

Solutions
What we are given in this problem is the conditional probability that a person approves of the President's new health care law based on the person's gender. In other words, we are given that:

$P(\text{Approve} \mid \text{Woman}) = 0.80 \text{ or } 80\%$
$P(\text{Approve} \mid \text{Man}) = 0.70 \text{ or } 70\%$

Based on the composition of the sample we can compute the probability a person is a man or a woman.

$P(\text{Man}) = \frac{200}{500} = 0.40 \text{ or } 40\% \quad \text{Answer to (2)}$
$P(\text{Woman}) = \frac{300}{500} = 0.60 \text{ or } 60\% \quad \text{Answer to (1)}$

Using the multiplication rule, $P(A \& B) = P(A \mid B) \times P(B)$, we can compute the four probabilities for: Men & Approve, Men & Disapprove, Women & Approve, Women & Disapprove.

The table below shows these four probabilities. The column totals show the total probability for "Approve" and "Disapprove".

<table>
<thead>
<tr>
<th></th>
<th>Approve</th>
<th>Disapprove</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>(0.70)(0.40) = 0.28</td>
<td>(0.30)(0.40) = 0.12</td>
<td>200/500 = 0.40</td>
</tr>
<tr>
<td>Women</td>
<td>(0.80)(0.60) = 0.48</td>
<td>(0.20)(0.60) = 0.12</td>
<td>300/500 = 0.60</td>
</tr>
<tr>
<td>Total</td>
<td>0.76</td>
<td>0.24</td>
<td>1</td>
</tr>
</tbody>
</table>
The probability that a person approves is:

\[ P(\text{Approve}) = P(\text{Approve} | \text{Man}) \cdot P(\text{Man}) + P(\text{Approve} | \text{Woman}) \cdot P(\text{Woman}) = \]

\[ = (.70)(.40) + (.80)(.60) = .28 + .48 = .76 \text{ or 76%} \text{ .......... Answer to (3)} \]

The probability that a person disapproves is:

\[ P(\text{Disapprove}) = 1 - P(\text{Approve}) = 1 - .76 = .24 \text{ or 24%} \text{ ................. Answer to (4)} \]

**Using a Tree Diagram**

Another type of diagram you can use to visualize this problem is a probability tree, shown below. Notice how the information we are given is used together with the multiplication rule to compute the four probabilities: Men & Approve, Men & Disapprove, Women & Approve, Women & Disapprove.