Section 7.1 Exercises

7.15 Finding the critical value \( t^* \). What critical value \( t^* \) from Table D should be used to calculate the margin of error for a confidence interval for the mean of the population in each of the following situations?

(a) A 95% confidence interval based on \( n = 11 \) observations.

(b) A 95% confidence interval from an SRS of 22 observations.

(c) A 90% confidence interval from a sample of size 22.

(d) These cases illustrate how the size of the margin of error depends upon the confidence level and the sample size. Summarize these relationships.

7.18 One-sided versus two-sided \( P \)-values. Computer software reports \( \bar{x} = 15.3 \) and \( P = 0.074 \) for a \( t \) test of \( H_0: \mu = 0 \) versus \( H_a: \mu \neq 0 \). Based on prior knowledge, you can justify testing the alternative \( H_a: \mu > 0 \). What is the \( P \)-value for your significance test?

7.19 More on one-sided versus two-sided \( P \)-values. Suppose that \( \bar{x} = -15.3 \) in the setting of the previous exercise. Would this change your \( P \)-value? Use a sketch of the distribution of the test statistic under the null hypothesis to illustrate and explain your answer.

7.20 A one-sample \( t \) test. The one-sample \( t \) statistic for testing

\[
H_0: \mu = 8 \\
H_a: \mu > 8
\]

from a sample of \( n = 16 \) observations has the value \( t = 2.10 \).

(a) What are the degrees of freedom for this statistic?

(b) Give the two critical values \( t^* \) from Table D that bracket \( t \).

(c) Between what two values does the \( P \)-value of the test fall?

(d) Is the value \( t = 2.10 \) significant at the 5% level? Is it significant at the 1% level?

(e) If you have software available, find the exact \( P \)-value.
7.21 Another one-sample t test. The one-sample t statistic for testing

\[ H_0: \mu = 40 \]
\[ H_a: \mu \neq 40 \]

from a sample of \( n = 28 \) observations has the value \( t = 2.01 \).

(a) What are the degrees of freedom for \( t \)?

(b) Locate the two critical values \( t^* \) from Table D that bracket \( t \).

(c) Between what two values does the \( P \)-value of the test fall?

(d) Is the value \( t = 2.01 \) statistically significant at the 5% level? At the 1% level?

(e) If you have software available, find the exact \( P \)-value.

7.22 A final one-sample t test. The one-sample t statistic for testing

\[ H_0: \mu = 20 \]
\[ H_a: \mu < 20 \]

based on \( n = 14 \) observations has the value \( t = -2.55 \).

(a) What are the degrees of freedom for this statistic?

(b) Between what two values does the \( P \)-value of the test fall?

(c) If you have software available, find the exact \( P \)-value.

7.24 Number of friends on Facebook. Facebook provides a variety of statistics on their Web site that detail the growth and popularity of the site. One such statistic is that the average user has 130 friends. Consider the following SRS of \( n = 30 \) Facebook users from a large university.

<table>
<thead>
<tr>
<th>99</th>
<th>148</th>
<th>158</th>
<th>126</th>
<th>118</th>
<th>112</th>
<th>103</th>
<th>111</th>
<th>154</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>127</td>
<td>137</td>
<td>74</td>
<td>85</td>
<td>104</td>
<td>106</td>
<td>72</td>
<td>119</td>
<td>160</td>
</tr>
<tr>
<td>83</td>
<td>110</td>
<td>97</td>
<td>193</td>
<td>96</td>
<td>152</td>
<td>105</td>
<td>119</td>
<td>171</td>
<td>128</td>
</tr>
</tbody>
</table>

(a) Do you think these data are Normally distributed? Use graphical methods to examine the distribution. Write a short summary of your findings.

(b) Is it appropriate to use the \( t \) methods of this section to compute a 95% confidence interval for the mean number of Facebook users at this large university? Explain why or why not.
(c) Find the mean, standard deviation, standard error, and margin of error for 95% confidence.

(d) Report the 95% confidence interval for \( \mu \), the average number of friends for Facebook users at this large university.

7.26 Fuel efficiency test. Computers in some vehicles calculate various quantities related to performance. One of these is the fuel efficiency, or gas mileage, usually expressed as miles per gallon (mpg). For one vehicle equipped in this way, the mpg were recorded each time the gas tank was filled, and the computer was then reset. Here are the mpg values for a random sample of 20 of these records:

\[
\begin{array}{cccccccccccc}
41.5 & 50.7 & 36.6 & 37.3 & 34.2 & 45.0 & 48.0 & 43.2 & 47.7 & 42.2 \\
43.2 & 44.6 & 48.4 & 46.4 & 46.8 & 39.2 & 37.3 & 43.5 & 44.3 & 43.3
\end{array}
\]

(a) Describe the distribution using graphical methods. Is it appropriate to analyze these data using methods based on Normal distributions? Explain why or why not.

(b) Find the mean, standard deviation, standard error, and margin of error for 95% confidence.

(c) Report the 95% confidence interval for \( \mu \), the mean mpg for this vehicle based on these data.

7.32 Food intake and weight gain. If we increase our food intake, we generally gain weight. Nutrition scientists can calculate the amount of weight gain that would be associated with a given increase in calories. In one study, 16 nonobese adults, aged 25 to 36 years, were fed 1000 calories per day in excess of the calories needed to maintain a stable body weight. The subjects maintained this diet for 8 weeks, so they consumed a total of 56,000 extra calories. According to theory, 3500 extra calories will translate into a weight gain of 1 pound. Therefore, we expect each of these subjects to gain 56,000/3500 = 16 pounds (lb). Here are the weights before and after the 8-week period expressed in kilograms (kg):

\[
\begin{array}{cccccccccc}
\text{Subject} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\text{Weight before} & 55.7 & 54.9 & 59.6 & 62.3 & 74.2 & 75.6 & 70.7 & 53.3 \\
\text{Weight after} & 61.7 & 58.8 & 66.0 & 66.2 & 79.0 & 82.3 & 74.3 & 59.3 \\
\text{Subject} & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
\text{Weight before} & 73.3 & 63.4 & 68.1 & 73.7 & 91.7 & 55.9 & 61.7 & 57.8 \\
\text{Weight after} & 79.1 & 66.0 & 73.4 & 76.9 & 93.1 & 63.0 & 68.2 & 60.3
\end{array}
\]

(a) For each subject, subtract the weight before from the weight after to determine the weight change.

(b) Find the mean and the standard deviation for the weight change.
(c) Calculate the standard error and the margin of error for 95% confidence. Report the 95% confidence interval in a sentence that explains the meaning of the 95%.

(d) Convert the mean weight gain in kilograms to mean weight gain in pounds. Because there are 2.2 kg per pound, multiply the value in kilograms by 2.2 to obtain pounds. Do the same for the standard deviation and the confidence interval.

(e) Test the null hypothesis that the mean weight gain is 16 lb. Be sure to specify the null and alternative hypotheses, the test statistic with degrees of freedom, and the P-value. What do you conclude?

(f) Write a short paragraph explaining your results.

7.34 Potential insurance fraud? Insurance adjusters are concerned about the high estimates they are receiving from Jocko’s Garage. To see if the estimates are unreasonably high, each of 10 damaged cars was taken to Jocko’s and to another garage and the estimates recorded. Here are the results:

(a) For each car, subtract the estimate of the other garage from Jocko’s estimate. Find the mean and the standard deviation for this difference.

(b) Test the null hypothesis that there is no difference between the estimates of the two garages. Be sure to specify the null and alternative hypotheses, the test statistic with degrees of freedom, and the P-value. What do you conclude using the 0.05 significance level?

(c) Construct a 95% confidence interval for the difference in estimates.

(d) The insurance company is considering seeking repayment from 1000 claims filed with Jocko’s last year. Using your answer to part (c), what repayment would you recommend the insurance company seek? Explain your answer.

7.39 Comparing operators of a DXA machine. Dual-energy X-ray absorptiometry (DXA) is a technique for measuring bone health. One of the most common measures is total body bone mineral content (TBBMC). A highly skilled operator is required to take the measurements. Recently, a new DXA machine was purchased by a research lab and two operators were trained to take the measurements. TBBMC for eight subjects was measured by both operators. The units are grams (g). A comparison of the means for the two operators provides a check on the training they received and allows us to determine if one of the operators is producing measurements that are consistently higher than the other. Here are the data:

<table>
<thead>
<tr>
<th>Car</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jocko's</td>
<td>1375</td>
<td>1550</td>
<td>1250</td>
<td>1300</td>
<td>900</td>
</tr>
<tr>
<td>Other</td>
<td>1250</td>
<td>1300</td>
<td>1250</td>
<td>1200</td>
<td>950</td>
</tr>
<tr>
<td>Car</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Jocko's</td>
<td>1500</td>
<td>1750</td>
<td>3600</td>
<td>2250</td>
<td>2800</td>
</tr>
<tr>
<td>Other</td>
<td>1575</td>
<td>1600</td>
<td>3300</td>
<td>2125</td>
<td>2600</td>
</tr>
</tbody>
</table>
(a) Take the difference between the TBBMC recorded for Operator 1 and the TBBMC for Operator 2. Describe the distribution of these differences.

(b) Use a significance test to examine the null hypothesis that the two operators have the same mean. Be sure to give the test statistic with its degrees of freedom, the \( P \)-value, and your conclusion.

(c) The sample here is rather small, so we may not have much power to detect differences of interest. Use a 95\% confidence interval to provide a range of differences that are compatible with these data.

(d) The eight subjects used for this comparison were not a random sample. In fact, they were friends of the researchers whose ages and weights were similar to the types of people who would be measured with this DXA. Comment on the appropriateness of this procedure for selecting a sample, and discuss any consequences regarding the interpretation of the significance testing and confidence interval results.

7.41 Assessment of a foreign-language institute. The National Endowment for the Humanities sponsors summer institutes to improve the skills of high school teachers of foreign languages. One such institute hosted 20 French teachers for 4 weeks. At the beginning of the period, the teachers were given the Modern Language Association’s listening test of understanding of spoken French. After 4 weeks of immersion in French in and out of class, the listening test was given again. (The actual French spoken in the two tests was different, so that simply taking the first test should not improve the score on the second test.) The maximum possible score on the test is 36. Here are the data:

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32</td>
<td>34</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
<td>31</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
<td>35</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>33</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
<td>36</td>
<td>3</td>
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<tr>
<td>7</td>
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</tr>
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<td>8</td>
<td>25</td>
<td>28</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
<td>26</td>
<td>-6</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>26</td>
<td>6</td>
</tr>
</tbody>
</table>

To analyze these data, we first subtract the pretest score from the posttest score to obtain the improvement for each teacher. These 20 differences form a single sample. They appear in the “Gain” columns. The first teacher, for example, improved from 32 to 34, so the gain is \( 34 - 32 = 2 \).
(a) State appropriate null and alternative hypotheses for examining the question of whether or not the course improves French spoken-language skills.

(b) Describe the gain data. Use numerical and graphical summaries.

(c) Perform the significance test. Give the test statistic, the degrees of freedom, and the $P$-value. Summarize your conclusion.

(d) Give a 95% confidence interval for the mean improvement.