Confidence Intervals & Hypothesis Testing About The Mean $\mu$ (mu):
Single Population With $\sigma$ (sigma) Given

Confidence Intervals

A level $C$ confidence interval for $\mu$ is given by:

$$\bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$$

Where $z^*$ is the upper $(1-C)/2$ critical $z$-value of the standard normal distribution. Values of $z^*$ for given values of $C$ (expressed as a %) are given in table D.

Using the TI-83/84: $z^* = |\text{invNorm}((1-C)/2)|$

Hypothesis Testing

**NULL HYPOTHESIS**
$H_0 : \mu = \mu_0$

**ALTERNATIVE HYPOTHESIS**

- $H_a : \mu > \mu_0$
- $H_a : \mu < \mu_0$
- $H_a : \mu \neq \mu_0$

**REJECT $H_0$ AT LEVEL $\alpha$ IF:**

- $z_{obs} \geq z^*_\alpha$
- $z_{obs} \leq -z^*_\alpha$
- $z_{obs} \leq -z^*_\alpha/2$ OR $z_{obs} \geq z^*_\alpha/2$
- $2P[Z \geq |z_{obs}|]$

**P-VALUE**

Notation and Conditions

1. The Z-statistic has the Standard Normal distribution, provided the sample is taken from a normal population OR approximately normal if $n \geq 30$.
2. $z_{obs}$ is the observed value of the statistic computed from the sample data.
3. $z^*_\alpha$ or $z^*_\alpha/2$ is the critical $z$-value based on the given $\alpha$ (one-sided alternative) or $\alpha/2$ (two-sided alternative). Using a TI-83/84: $z^*_\alpha = |\text{invNorm}(\alpha)|$. 
Confidence Intervals &
Hypothesis Testing About The Mean \( \mu(\text{mu}) \):
Single Population With \( \sigma(\text{sigma}) \) Not Given

Confidence Intervals

A level C confidence interval for \( \mu \) is given by:

\[
\bar{x} \pm t^* \left( \frac{S}{\sqrt{n}} \right)
\]

Where \( t^* \) is the upper \((1-C)/2\) critical t-value of the student t-distribution with \( n-1 \) degrees of freedom. Values of \( t^* \) for given values of \( C \) (expressed as a \%) are given in table D.

Using the TI-84: \( t^* = |\text{invT}((1-C)/2, \text{df})| \).
TI-83 does not have this function, but for values of \( n > 30 \), \( t^* \) and \( z^* \) are approximately equal.

Hypothesis Testing

<table>
<thead>
<tr>
<th>NULL HYPOTHESIS</th>
<th>ALTERNATIVE HYPOTHESIS</th>
<th>REJECT H, AT LEVEL</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 : \mu = \mu_0 )</td>
<td>( H_a : \mu &gt; \mu_0 )</td>
<td>( t_{\text{obs}} \geq t^*_{\alpha} )</td>
<td>( P[T \geq t_{\text{obs}}] )</td>
</tr>
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<td>( H_0 : \mu &lt; \mu_0 )</td>
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</tr>
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</table>

Notation and Conditions

1. The T-statistic has the student t-distribution with \( df = n - 1 \) provided random sample is from a normal population OR \( n \geq 30 \)
2. \( t_{\text{obs}} \) is the observed value of the statistic, which is computed from the sample data.
3. \( t^*_{\alpha} \) or \( t^*_{\alpha/2} \) is the critical t-value based on the given \( \alpha \) (one-sided alternative) or \( \alpha/2 \) (two-sided alternative).
Comparing The Means of Two Populations: Two Independent Samples

Confidence Intervals

A level C confidence interval for $\mu_1 - \mu_2$ is given by:

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Where $t^*$ is the upper (1-C)/2 critical t-value of the student t-distribution with degrees of freedom the smaller of $n_1 - 1$ or $n_2 - 1$. Values of $t^*$ for given values of C (expressed as a %) are given in table D.

Hypothesis Testing

**NULL HYPOTHESIS**  
$H_0 : \mu_1 = \mu_2$

**ALTERNATIVE HYPOTHESIS**  
$H_A : \mu_1 > \mu_2$  
$H_A : \mu_1 < \mu_2$  
$H_A : \mu_1 \neq \mu_2$

**REJECT H. AT LEVEL $\alpha$ IF:**  
$t_{obs} \geq t^*_\alpha$  
$t_{obs} \leq -t^*_\alpha$  
$t_{obs} \leq -t^*_{\alpha/2}$ OR  
$t_{obs} \geq t^*_{\alpha/2}$

**P-VALUE**

$P[T \geq t_{obs}]$  
$P[T \leq -t_{obs}]$  
$2P[T \geq |t_{obs}|]$

Notation and Conditions

1. The T-statistic has the student t-distribution with $df^* = \text{the smaller of } n_1 - 1 \text{ or } n_2 - 1$, provided the samples come from two normal populations (or that eache sample size is greater than 30) normal population OR $n \geq 30$

2. $t_{obs}$ is the observed value of the statistic, which is computed from the sample data.

3. $t^*_\alpha$ or $t^*_{\alpha/2}$ is the critical t-value based on the given $\alpha$ (one-sided alternative) or $\alpha/2$ (two-sided alternative).

(*) When the sample sizes are large, the two sample t-procedures are fairly accurate with $df = \text{the smaller of } n_1 - 1 \text{ or } n_2 - 1$. With small sample sizes it is recommended that $t^*_\alpha$ be computed using the t-distribution with degrees of freedom calculated using the formula:
Confidence Intervals for $p$ - One Sample

**Note:** $p$ here stands for the true proportion (fraction, also expressed as %) of all members in a population of a particular attribute. $p$ is most commonly known as the *population proportion*.

<table>
<thead>
<tr>
<th>Sample Statistic</th>
<th>Standard Error of the Sample Statistic</th>
<th>Level C Confidence Interval</th>
</tr>
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<tbody>
<tr>
<td>$\hat{p} = \frac{X}{n}$</td>
<td>$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$</td>
<td>$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$</td>
</tr>
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</table>

Note: $z^* = z_{\alpha/2}$ (where $\alpha = 1 - C$)

**Sample Size For a Margin of Error $m$ For Estimating $p$**

The level C confidence interval for the population proportion $p$ will have margin of error at most $m$, if $n$ is chosen to be:

$$n = \left( \frac{z^*}{2m} \right)^2$$

**Hypotheses Testing for $p$ - One Sample**

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</table>
Assumptions:

1. The sample must come from a large population
2. np > 5 and n(1-p) > 5

Notation and Conditions

1. The **Z-statistic** has the Standard Normal distribution
2. $z_{obs}$ is the observed value of the statistic, which is computed from the sample data.
3. $z^*_{\alpha}$ or $z^*_{\alpha/2}$ is the critical z-value based on the given $\alpha$ (one-sided alternative) or $\alpha/2$ (two-sided alternative).